

# Composite fermion edge states, fractional charge and current noise

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## Abstract

A composite fermion edge state theory of current fluctuations, fractional quasiparticle charge and Johnson-Nyquist noise in the fractional quantum Hall regime is presented. It is shown that composite fermion current fluctuations and the charges of the associated quasiparticles are strongly renormalized by the interactions between composite fermions. The important interaction is that mediated by the fictitious electric field associated with composite fermion currents. The dressed current fluctuations and quasiparticle charges are calculated self-consistently in a mean field theory for smooth edges. Analytic results are obtained. The values of the fractional quasiparticle charges obtained agree with the predictions of previous theories in the incompressible regions of the 2DEG where those theories apply. In the compressible regions the magnitudes of the quasiparticle charges vary with position. Since Johnson-Nyquist noise arises from the compressible regions, it is due to quasiparticles whose charges differ from the simple fractions of  $e$  that apply in the incompressible regions. Never the less, the Nyquist noise formula  $S = 4k_B T G$  is obeyed on fractional quantum Hall plateaus. Some implications for the interpretation of recent shot noise measurements in the fractional quantum Hall regime are briefly discussed.

PACS: 73.40.Hm, 73.50.Td

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## I. INTRODUCTION

It was first shown by Laughlin,<sup>1</sup> that the fractional quantum Hall effect occurs because electron-electron interactions result in incompressible states of two-dimensional electron systems at special filling fractions of a Landau level. Subsequently, Jain showed that these incompressible many-body states can be understood, in a mean field sense, as arising from the spectral gaps between the single-particle Landau levels of quasi-particles known as composite fermions and that the fractional quantum Hall effect can be viewed as the integer quantum Hall effect of composite fermions.<sup>2</sup> The composite fermion theory has yielded many remarkable results and there is now an extensive theoretical and experimental literature supporting it and examining its ramifications.<sup>3</sup>

Following Jain's suggestion that the fractional quantum Hall effect is the integer quantum Hall effect of composite fermions, a model of composite fermion edge states has been proposed<sup>4,5</sup> that generalizes the very successful edge state theories of transport in the integer quantum Hall regime<sup>6-9</sup> to fractional quantum Hall phenomena. This mean field model was developed for high mobility systems with smooth edge potentials and has been able to account for a great deal of experimental data including the observed integer<sup>10</sup> and fractional<sup>11</sup> quantized Hall conductances, the results of transport experiments on Hall bars with smooth potential barriers and constrictions,<sup>12-18</sup> and the observed Fermi liquid-like behavior of Aharonov-Bohm resonances associated with antidots in the fractional quantum Hall regime.<sup>19,20</sup>

The singular gauge transformation<sup>2,21,22</sup> that transforms electrons into composite fermions by attaching to them tubes of fictitious magnetic flux preserves charge, and therefore the charge of a composite fermion is equal to that of an electron. However, it was pointed out by Goldhaber and Jain<sup>23</sup> that the local charge associated with the composite fermion is dressed by the presence of the other composite fermions in the system and thus becomes fractional, the fractions being in agreement with the fractional quasiparticle charges predicted earlier by Laughlin and others.<sup>1,24,25</sup> For example, for the fractional filling  $\nu = 1/3$  of the lowest Landau level the dressed quasiparticle charge  $-e^*$  equals  $-e/3$ .

Recently experiments have been reported measuring current noise in the fractional quantum Hall regime<sup>26,27</sup> as a probe of the fractional quasiparticle charge. The results have been interpreted<sup>26,27</sup> as direct experimental evidence of fractionally charged quasiparticles with  $e^* = e/3$  at  $\nu = 1/3$ . However, the theoretical predictions<sup>1,23-25</sup> that  $e^* = e/3$  at  $\nu = 1/3$  are based on the assumption that the electronic state is *incompressible*, whereas current noise originates at the edges of the sample where both incompressible and compressible regions occur. Furthermore Johnson-Nyquist noise arises *entirely* from the *compressible* regions. It is therefore of interest to examine both the fractional quasiparticle charges and the associated current noise theoretically within an edge state model that admits both incompressible and compressible regions at the edge. This is the purpose of the present article.

Section II contains a brief summary of those aspects of composite fermion mean field theory and of the edge state model<sup>4,5</sup> that will be used in the remainder of this paper.

In Section III I discuss electric current fluctuations in the fractional quantum Hall regime within the framework of the edge state model,<sup>4,5</sup> and show that to understand these fluctuations one must consider the interactions between the composite fermions, the important interactions being those mediated by the fictitious electric field<sup>28,29,30,23</sup> associated with

composite fermion currents. These interactions renormalize the current fluctuations and their effects must be calculated self-consistently. This is done analytically in a mean field approximation and it is found that the dressing of the current fluctuations can be interpreted as a dressing of the charges of the composite fermion quasi-particles. The dressed composite fermion charges are fractional and in incompressible regions the predicted fractions agree with those obtained earlier by Laughlin and others.<sup>1,24,25,23</sup> However in the compressible regions near the edge the fractions are predicted to be different and to vary smoothly with position. As the edge is approached and the local electron density tends to zero the dressed composite fermion charge approaches the electron charge  $-e$ .

Current noise in mesoscopic systems has been discussed theoretically in recent years by many authors,<sup>31–39</sup> but not from the perspective of composite fermion edge states. In Section IV I calculate the Johnson-Nyquist noise on the fractional quantum Hall plateaus using the composite fermion edge state model<sup>4,5</sup> and the results of Section III for the dressed current fluctuations. The method used is a generalization of the wave packet arguments of Landauer and Martin<sup>32,35</sup> to composite fermion edge state theory and the fractional quantum Hall regime. Summing the contributions of all of the active and silent modes<sup>4</sup> of the edge state model and including the variation of the dressed quasiparticle charge across the edge obtained in Section III yields a result in agreement with the Nyquist formula  $S = 4k_B T G_H$  for all of Jain’s filling fractions of a Landau level. Here  $S$  is the Johnson-Nyquist noise on the fractional quantum Hall plateau,  $k_B$  is Boltzmann’s constant,  $T$  is the temperature and  $G_H$  is the fractional quantized Hall conductance.

The significance of these results is discussed in Section V where I also comment briefly on the implications for the interpretation of recent shot noise experiments.

## II. EDGE STATE MODEL

In composite fermion theory, a singular gauge transformation attaches a tube of fictitious magnetic flux with an even number of flux quanta to each electron.<sup>2,21,22</sup> The electrons together with the attached flux tubes obey Fermi statistics and are called “composite fermions.”<sup>2</sup> In mean field theory the interactions between composite fermions that are due to the vector potentials associated with the tubes of fictitious flux are replaced by interactions with a fictitious average magnetic field and a fictitious electric field.

The fictitious average magnetic field is given by<sup>2,21,22</sup>

$$\mathbf{B}_g = -n_e \hat{\mathbf{B}} m \frac{h}{e} = -m\nu \mathbf{B} \quad (1)$$

where an even integer number  $m$  of flux quanta  $h/e$  are attached to each electron,  $n_e$  is the two-dimensional electron density,  $\hat{\mathbf{B}}$  is the unit vector pointing in the direction of the true magnetic field  $\mathbf{B}$  and  $\nu = n_e h/eB$  is the Landau level filling parameter. Thus the composite fermions experience an effective magnetic field

$$\mathbf{B}_{eff} = \mathbf{B} + \mathbf{B}_g \quad (2)$$

and fill effective Landau levels spaced in energy by  $\hbar\omega_{eff}$  where

$$\omega_{eff} = e|\mathbf{B}_{eff}|/m^* \quad (3)$$

is the effective cyclotron frequency and  $m^*$  is the composite fermion effective mass. The fictitious electric field is<sup>28,29,30,23</sup>

$$\mathbf{E}_g = -(\mathbf{J} \times \hat{\mathbf{B}})mh/e^2 \quad (4)$$

where  $\mathbf{J}$  is the two-dimensional electric current density.

In the composite fermion edge state model<sup>4,5</sup> it is assumed that the electron density is slowly varying with position and that, for local Landau level filling fractions in the vicinity of  $1/m$ , the composite fermion Landau level energies behave qualitatively like

$$E_{m,r} = \left(r + \frac{1}{2}\right) \hbar\omega_{eff} + W \quad (5)$$

where  $r = 0, 1, 2, \dots$  and  $W$  is the position-dependent composite fermion effective potential energy that includes the effects of the fictitious electric field. Equation (5) is a good description of the composite fermion Landau level structure in uniform systems<sup>2</sup> and yields edge states that propagate in the direction consistent with experiments.<sup>4,40</sup> This behavior of the composite fermion Landau levels near an edge is illustrated in Fig. 1. The effective magnetic field  $\mathbf{B}_{eff}$  well away from the edge is parallel to the real magnetic field in Fig.1(a) and anti-parallel in Fig.1(b). The apex of each “fan” of energy levels occurs where  $\mathbf{B}_{eff} = 0$  for some even integer  $m$ ; there according to equations (1) and (2)  $E_{m,r} = W$ . The different types of composite fermion Landau levels are labeled I, II and III. Type I Landau levels are silent edge modes in the sense that the average currents that they carry are independent of the electrochemical potential at the edge, provided that quasi-equilibrium conditions prevail there.<sup>4</sup> However, Landau levels of types II and III carry non-zero average net currents when the difference between the composite fermion effective electrochemical potentials at opposite edges of the sample is not zero.<sup>4</sup>

### III. CURRENT FLUCTUATIONS AND FRACTIONAL CHARGE

Consider a 2DEG of length  $L$  in the  $x$ - $y$  plane connecting source and drain contacts as depicted in Fig.2. Suppose that the magnetic field  $\mathbf{B}$  points in the  $z$ -direction and define an effective vector potential  $\mathbf{A}_{eff} = (0, \int^x B_{eff}(u)du, 0)$  such that the effective magnetic field experienced by composite fermions is given by  $\mathbf{B}_{eff} = \nabla \times \mathbf{A}_{eff}$ . The effective composite fermion Schrödinger equation for the  $r^{th}$  composite fermion Landau level is then<sup>41</sup>

$$\left(\frac{1}{2m^*}\left(\frac{\hbar}{i}\nabla + e\mathbf{A}_{eff}\right)^2 + W(x)\right)\psi_{r,k}(x, y) = \epsilon_{r,k}\psi_{r,k}(x, y) \quad (6)$$

where

$$\psi_{r,k}(x, y) = e^{iky}X_{r,k}(x). \quad (7)$$

The net electric current in the  $y$ -direction is then

$$I = -\frac{1}{L} \sum_{r,k} e N_{r,k} v_{r,k} \quad (8)$$

where  $N_{r,k}$  and  $v_{r,k}$  are the occupation and velocity of the single particle composite fermion state  $|r, k\rangle$  and  $L$  is the length of the sample. Since composite fermions obey Fermi statistics,  $N_{r,k}$  takes the values 0 and 1. Fluctuations in the values that the  $N_{r,k}$  take give rise to current noise. However an important complication is that  $\epsilon_{r,k}$  in Equation (6) (and therefore  $v_{r,k}$  in Equation (8)) depends on the specific configuration of occupation numbers  $N_{r,k}$  as will be discussed further below.

In the spirit of mean field theory let us now approximate the average electric current  $\langle I \rangle$  by setting every occupation number  $N_{r,k}$  in Equation (8) formally equal to its ensemble average value  $\langle N_{r,k} \rangle$  and replacing  $v_{r,k}$  by the corresponding mean field velocity function  $v_{r,k}^{av} = \frac{1}{\hbar} \frac{d\epsilon_{r,k}^{av}}{dk}$ , where  $\epsilon_{r,k}^{av}$  is the solution of Equation (6) with  $\mathbf{A}_{eff}$  and  $W$  calculated using the average occupation numbers  $\langle N_{r,k} \rangle$ . Then

$$\langle I \rangle = -\frac{1}{L} \sum_{r,k} e \langle N_{r,k} \rangle v_{r,k}^{av} \quad (9)$$

or equivalently

$$\langle I \rangle = -\frac{e}{h} \sum_r \int \langle N_{r,k} \rangle d\epsilon_{r,k}^{av} \quad (10)$$

Now let us alter the occupation number  $N_{R,K}$  of just *one* single-particle state  $|R, K\rangle$  so that it differs from the mean  $\langle N_{R,K} \rangle$  that figures in Equations (9) and (10), and examine the corresponding deviation  $\delta I_{R,K}$  of the current  $I$  from  $\langle I \rangle$ . (General current fluctuations will be treated at the end of this Section.)

Bearing in mind Equation (10),  $\delta I_{R,K}$  can be written as

$$\delta I_{R,K} = \delta I_{R,K}^0 - \frac{e}{h} \sum_r \int \langle N_{r,k} \rangle dU_{r,k}. \quad (11)$$

Here

$$U_{r,k} = \epsilon_{r,k}^* - \epsilon_{r,k}^{av} \quad (12)$$

and

$$\delta I_{R,K}^0 = -\frac{e}{L} (N_{R,K} - \langle N_{R,K} \rangle) v_{R,K}^* \quad (13)$$

where  $\epsilon_{r,k}^*$  and  $v_{r,k}^*$  are the self-consistent single particle energies and velocities for the the composite fermion system with only  $N_{R,K}$  differing from its mean.

Notice that the second term on the RHS of Equation (11) includes the effect of changing the occupation of the single particle composite fermion state in Landau Level R with wave vector K on the currents carried by the *other* single particle composite fermion states. It will be seen below that this effect is very important.

To proceed further analytically I will assume the edge currents are of the quasi-equilibrium type and that the edge potential  $W(x)$  is so slowly varying with  $x$  that  $\langle N_{r,k} \rangle$

varies little over the range of values of  $k$  that contribute appreciably<sup>42</sup> to the integral in Equation (11). In Equation (11),  $\int < N_{r,k} > dU_{r,k}$  then becomes  $< N_{r,K} > \int dU_{r,k}$ .

The occupation number fluctuation  $N_{R,K} - < N_{R,K} >$  affects  $U_{r,k}$  through the associated fluctuations in the fictitious electric field  $\mathbf{E}_g$ , the Coulomb potential and the effective magnetic field. Only the first of these contributes to  $\int dU_{r,k}$  because the others are local effects and do not affect the integral. Thus using Equation (4) for  $\mathbf{E}_g$  yields

$$\int dU_{r,k} = \Delta U = \int e \delta \mathbf{E}_g \cdot d\mathbf{x} = \pm \frac{mh}{e} \delta I_{R,K} \quad (14)$$

where the upper (lower) sign applies when  $\mathbf{B}_{eff}$  is parallel (antiparallel) to  $\mathbf{B}$ . Notice that the *total* current fluctuation  $\delta I_{R,K}$  is used for the integral of the fluctuation current density in the last term in Equation (14). This ensures that the final result (16) of this calculation will include the interactions between composite fermions that are mediated by the fluctuation  $\delta \mathbf{E}_g$  of the fictitious electric field *self-consistently*.

Replacing  $\int < N_{r,k} > dU_{r,k}$  by  $< N_{r,K} > \int dU_{r,k}$  in Equation (11) and using (14) yields

$$\delta I_{R,K} = \delta I_{R,K}^0 \mp m \delta I_{R,K} \sum_r < N_{r,K} > \quad (15)$$

or

$$\delta I_{R,K} = \frac{\delta I_{R,K}^0}{1 \pm m \sum_r < N_{r,K} >} \quad (16)$$

The physical meaning of this result is that a single-particle composite fermion current fluctuation  $\delta I_{R,K}^0$  generates a fluctuation in the fictitious electric field that extends over a distance of a few magnetic lengths. As a result a “kink” develops in the energy dispersions of *all* of the composite fermion Landau levels in that vicinity modifying the velocities of the surrounding composite fermion states. Thus the single particle current fluctuation is dressed by the disturbance that it generates in the surrounding composite fermion medium. The dressing must be calculated self-consistently since the current fluctuation induced in the surrounding medium generates its own contribution to the fluctuation in the fictitious electric field which in turn affects the current. The self-consistent calculation yields the dressed current fluctuation  $\delta I_{R,K}$  given by Equation (16).

Thus the current fluctuation can be understood as a dressed quasi-particle associated with the composite fermion in Landau level R with wave vector K. The quasi-particle propagates with same velocity  $v_{R,K}^*$  as the composite fermion. It is therefore intuitively appealing to rewrite equation Equation (16) for the dressed current fluctuation in a form analogous to Equation (13) for the “bare” current fluctuation but with a renormalized quasi-particle charge, i.e.,

$$\delta I_{R,K} = -\frac{e_{R,K}^*}{L} (N_{R,K} - < N_{R,K} >) v_{R,K}^* \quad (17)$$

where the dressed quasi-particle charge is given by

$$-e_{R,K}^* = \frac{-e}{1 \pm m \sum_r < N_{r,K} >} \quad (18)$$

Where the 2DEG is incompressible,  $\langle N_{r,K} \rangle$  is either 0 or 1 depending on whether the  $r^{th}$  composite fermion Landau level is full or empty, and therefore

$$-e^* = \frac{-e}{1 \pm mn} \quad (19)$$

where  $n = \sum_r \langle N_{r,K} \rangle$  is an integer and  $m$  is an even integer. This agrees with the fractional quasi-particle charges that have been derived previously by other methods for incompressible states at Jain's filling fractions  $\nu = \frac{n}{mn \pm 1}$  of a Landau level.<sup>1,24,25,23</sup> For example, in the bulk for the fractional filling  $\nu = 1/3$  of the lowest Landau level,  $m = 2$ ,  $n = 1$  and the dressed quasiparticle charge  $-e^*$  given by Equation (19) equals  $-e/3$ .

On the other hand as the depletion region at the edge of the sample is approached the filling of the composite fermion Landau levels approaches zero. Thus  $\sum_r \langle N_{r,K} \rangle \rightarrow 0$  and Equation (18) predicts that  $-e^*$  tends to  $-e$ .

In the compressible regions between these two extremes the behavior of the quasi-particle charge can be obtained from Equation (18) if the behavior of  $\langle N_{r,k} \rangle$  is known. For the usual case of quasi-equilibrium edge currents this is given by the Fermi function

$$\langle N_{r,k} \rangle = \frac{1}{e^{\beta(\epsilon_{r,k}^{av} - \mu_i^*)} + 1} \quad (20)$$

where  $\mu_i^*$  is the composite fermion electrochemical potential for the  $i^{th}$  (closest) edge,  $\beta = 1/k_B T$ ,  $T$  is the temperature and  $k_B$  is Boltzmann's constant. Johnson-Nyquist noise arises from the compressible regions in this regime and will be evaluated using the above results in Section IV.

Since the modification of the velocities of the composite fermions from  $v_{r,k}^{av}$  to  $v_{r,k}^*$  by the fictitious electric field associated with the current fluctuation is responsible for the dressing of the composite fermion charge and since the dressed velocity  $v_{R,K}^*$  appears explicitly in Equation (17) along with  $e^*$  it is of interest to consider how large the effect of the fluctuation of the fictitious electric field on  $v_{R,K}^*$  may be. This may be estimated as  $\sim \frac{\Delta U}{\hbar \Delta k}$  where  $\Delta U$  is as in Equation (14) and  $\Delta k$  is the difference in wave vector between states separated by the magnetic length  $a = \hbar/(eB_{eff})$  in real space. The result is found to be proportional to  $a/L$  and is therefore very small for macroscopic samples. The dressing of the quasi-particle charge on the other hand is due to the modification by the fictitious electric field of the velocities of a *large* number (of the order of  $L/a$ ) of single-particle composite fermion states and for that reason is a very significant effect even though the modification of the individual composite fermion velocities is very small.

Finally, it is important to consider whether the above results which have been obtained by considering the dressing of a *single* single-particle current fluctuation are applicable to thermal current fluctuations which involve large numbers of such single-particle "excitations" simultaneously. This will be done in two steps:

Firstly, if a current fluctuation is due to any number of single particle excitations that are widely separated from each other (by much more than a magnetic length) in real space then the dressing of each single particle current fluctuation will occur separately from the others and it follows that Equation (17) becomes

$$\delta I = - \sum_{R,K} \frac{e_{R,K}^*}{L} (N_{R,K} - \langle N_{R,K} \rangle) v_{R,K}^* \quad (21)$$

where  $\delta I$  is the total dressed current fluctuation, the sum over  $R$  and  $K$  is over the individual single-particle excitations that contribute to the current fluctuation and the dressed composite fermion quasiparticle charge  $-e_{R,K}^*$  is given by Equation (18).

Secondly, if a current fluctuation is due to any number of single particle excitations that are all so close to each other in real space that  $\langle N_{r,k} \rangle$  does not vary appreciably over the entire region containing these excitations for any composite fermion Landau level  $r$  then the entire argument leading to Equation (16) still holds if  $\delta I_{R,K}^0$  is replaced with the sum of the “bare” single particle current fluctuations  $-\frac{1}{L} \sum_{R,K} e(N_{R,K} - \langle N_{R,K} \rangle) v_{R,K}^*$  and  $\delta I_{R,K}$  is replaced with the total dressed current fluctuation  $\delta I$ . The result of this calculation turns out to be also given by Equations (21) and (18).

Since Equations (21) and (18) are valid in the two opposite limits of widely separated and closely spaced single-particle excitations discussed above it is reasonable to suppose that they are valid (or at least a good approximation) in general, at the level of mean field theory and provided that the edge potential  $W(x)$  and  $\langle N_{r,k} \rangle$  are sufficiently slowly varying functions of  $x$  and  $k$  as has been assumed in the above analysis.

#### IV. JOHNSON-NYQUIST NOISE

I will now apply the theory developed above to the simplest relevant experimentally observable phenomenon, the Johnson-Nyquist noise on fractional quantum Hall plateaus where there is no backscattering of the composite fermions.

Johnson-Nyquist noise is the electric current noise that occurs in the absence of an applied voltage across a conductor connecting two reservoirs and is due to thermal fluctuations in the number of charge carriers that pass through the conductor per unit time. As depicted schematically in Fig. 2, the reservoirs are assumed to be effectively short circuited via two large capacitances to eliminate any voltage fluctuations between them at the frequencies of interest.

Current noise is described in terms of the spectral density of current-current fluctuations that is defined as follows.<sup>43</sup> Let the Fourier transform of the current fluctuations be

$$\delta I_\omega = \int_{-\infty}^{\infty} \delta I(t) e^{i\omega t} dt. \quad (22)$$

The current-current correlation function  $\langle \delta I(t) \delta I(t') \rangle$  depends only on  $t - t'$  if  $\langle \delta I_\omega \delta I_{\omega'} \rangle$  is of the form

$$\langle \delta I_\omega \delta I_{\omega'} \rangle = 2\pi F_\omega \delta(\omega + \omega') \quad (23)$$

It then follows that  $\langle (\delta I)^2 \rangle \equiv \langle \delta I(t) \delta I(t) \rangle$  is given by

$$\langle (\delta I)^2 \rangle = \frac{1}{\pi} \int_0^\infty F_\omega d\omega = 2 \int_0^\infty F_{2\pi f} df \quad (24)$$

where  $\omega = 2\pi f$ . Thus the spectral density of current fluctuations at frequency  $f$  is

$$S(f) = 2F_{2\pi f} \quad (25)$$

where  $F$  is defined by Equation (23).



Let us now rewrite Equation (21) as

$$\delta I(t) = \sum_{R,K} \delta I_{R,K}(t) \quad (26)$$

where  $\delta I_{R,K}(t)$  is the dressed contribution of the state with wave vector  $K$  in composite fermion Landau Level  $R$  at time  $t$  to the current fluctuation. The time dependence arises because the occupation number  $N_{R,K}$  fluctuates taking values 0 and 1 at different times as composite fermion wave packets in composite fermion Landau level  $R$  and centered on wave vector  $K$  are injected into the 2DEG and absorbed from it by the reservoirs. In the spirit of Landauer and Martin's treatment of systems of ordinary fermions,<sup>32,35</sup> I will assume that these wave packets (which may be occupied or empty) are mutually orthogonal and enter the 2DEG in a regular sequence at times  $t_{R,K,l}$  spaced by the time  $\delta t_{R,K} = t_{R,K,l+1} - t_{R,K,l}$  that it takes a wave packet to traverse a distance  $D$  equal to its length. For the low frequencies  $\omega$  of interest ( $\omega \ll 1/\delta t_{R,K}$ ), Equation (22) thus becomes

$$\delta I_\omega = \sum_{R,K} \int_{-\infty}^{\infty} \delta I_{R,K}(t) e^{i\omega t} dt = \sum_{R,K,l} \delta t_{R,K} \delta I_{R,K,l} e^{i\omega t_{R,K,l}} \quad (27)$$

where

$$\delta I_{R,K,l} = \frac{1}{\delta t_{R,K}} \int_{t_{R,K,l}}^{t_{R,K,l+1}} \delta I_{R,K}(t) dt. \quad (28)$$

Assuming that the current fluctuations for different  $R$ ,  $K$  and  $l$  are uncorrelated, Equation (27) yields

$$\langle \delta I_\omega \delta I_{\omega'} \rangle = \sum_{R,K,l} \langle \delta I_{R,K,l}^2 \rangle \delta t_{R,K} e^{i(\omega+\omega')t_{R,K,l}} \quad (29)$$

Noting that  $\langle \delta I_{R,K,l}^2 \rangle$  is independent of  $l$  and that for low frequencies  $\omega, \omega' \ll 1/\delta t_{R,K}$ , we may replace  $\sum_l e^{i(\omega+\omega')t_{R,K,l}}$  by  $2\pi\delta(\omega+\omega')/\delta t_{R,K}$ , Equation (29) becomes

$$\langle \delta I_\omega \delta I_{\omega'} \rangle = 2\pi \sum_{R,K} \langle \delta I_{R,K}^2 \rangle \delta t_{R,K} \delta(\omega+\omega') \quad (30)$$

which by comparison with Equations (23) and (25) yields

$$S(f) = 2 \sum_{R,K} \langle \delta I_{R,K}^2 \rangle \delta t_{R,K} \quad (31)$$

for the spectral density of the current noise.

In Equation (31)  $\delta I_{R,K}$  is as in Equations (17) and (21) but with  $L$  replaced by  $D$  since the wave packet basis is now being used. Setting  $\langle N_{R,K}^2 \rangle = \langle N_{R,K} \rangle$  since  $N_{R,K} = 0$  or  $1$ , using  $|v_{R,K}^*| = D/\delta t_{R,K}$  and approximating  $v_{R,K}^*$  by  $v_{R,K}^{av}$  (which is reasonable according to the discussion in Section III), Equation (31) yields

$$S(f) = 2 \sum_{R,K} \frac{e_{R,K}^{*2}}{D} \langle N_{R,K} \rangle (1 - \langle N_{R,K} \rangle) |v_{R,K}^{av}| \quad (32)$$

Note that since  $0 \leq \langle N_{R,K} \rangle \leq 1$  the summand in Equation (32) is non-negative and is not zero only for states in compressible regions. Transforming the sum over  $K$  in to an energy integral yields

$$S(f) = \frac{2}{h} \sum_R \int e_{R,K}^{*2} \langle N_{R,K} \rangle (1 - \langle N_{R,K} \rangle) d\epsilon_{R,K}^{av} \quad (33)$$

Assuming that  $\langle N_{R,K} \rangle$  is of the form given by Equation (20) yields

$$\frac{\partial \langle N_{R,K} \rangle}{\partial \epsilon_{R,K}^{av}} = -\beta \langle N_{R,K} \rangle (1 - \langle N_{R,K} \rangle) \quad (34)$$

and Equation (33) becomes

$$S(f) = \frac{2}{h\beta} \sum_R \int e_{R,K}^{*2} d\langle N_{R,K} \rangle. \quad (35)$$

Finally, noting the form (18) of the dressed composite fermion charge, Equation (35) reduces to

$$S(f) = \frac{2e^2}{h\beta} \int \frac{dN_{\text{tot}}}{(1 \pm mN_{\text{tot}})^2} \quad (36)$$

where  $N_{\text{tot}}$  stands for  $\sum_r \langle N_{r,K} \rangle$ , the local total filling of the composite fermion Landau levels. In keeping with the non-negative character of the summand in Equation (32), the convention has been adopted that the direction of integration in Equations (33), (35) and (36) is such that  $d\epsilon_{R,K}^{av}$ ,  $d\langle N_{R,K} \rangle$  and  $dN_{\text{tot}}$  are always positive, respectively.

Equation (36) gives the low frequency Johnson-Nyquist noise on fractional quantum Hall plateaus where there is no back scattering of composite fermions. In general the integral consists of separate contributions from each region where  $\mathbf{B}_{eff}$  is parallel or antiparallel to  $\mathbf{B}$  at each of the edges of the 2DEG that run between the source and drain in Fig.2.

Let us consider now the case illustrated in Fig.1(a) where  $\mathbf{B}_{eff}$  is parallel to  $\mathbf{B}$  far from the edges where the Landau level filling  $\nu = n_b/(m_b n_b + 1)$  is a Jain fraction with  $n_b$  a positive integer and  $m_b$  an even positive integer.

Suppose initially for simplicity that only the Type III composite fermion Landau levels are present and that they empty one by one as the edge is approached. Then  $N_{\text{tot}}$  in Equation (36) ranges from 0 at the edge to  $n_b$ , the number of occupied composite fermion levels in the bulk, and the Johnson-Nyquist noise (including the contributions from both edges of the 2DEG) is given by

$$S(f) = \frac{4e^2}{h\beta} \int_0^{n_b} \frac{dN_{\text{tot}}}{(1 + m_b N_{\text{tot}})^2} = \frac{4}{\beta} \frac{e^2}{h} \frac{n_b}{(m_b n_b + 1)} \quad (37)$$

This is just the Nyquist formula  $S(f) = 4k_B T G$  since on a quantum Hall plateau the two-terminal conductance  $G$  is equal to the quantized Hall conductance  $G_H = \frac{e^2}{h} \frac{n_b}{(m_b n_b + 1)}$ .

Now let us consider the contribution of the “silent”<sup>4</sup> Type I composite fermion Landau levels in Fig.1(a) to  $S(f)$ . For these  $m = m_b + 2$ . Let us suppose that the crossover from

the Type III to the Type I states occurs at a local Landau level filling fraction  $\nu_c$  where  $N_{\text{tot}} = n_c$  for the Type III states and at  $N_{\text{tot}} = n'_c$  for the Type I states. Then

$$\frac{n_c}{m_b n_c + 1} = \nu_c = \frac{n'_c}{(m_b + 2)n'_c - 1} \quad (38)$$

and

$$S(f) = \frac{4e^2}{h\beta} \left( \int_{n_c}^{n_b} \frac{dN_{\text{tot}}}{(1 + m_b N_{\text{tot}})^2} + \int_{n'_c}^{\infty} \frac{dN_{\text{tot}}}{(1 - (m_b + 2)N_{\text{tot}})^2} + \int_0^{\infty} \frac{dN_{\text{tot}}}{(1 + (m_b + 2)N_{\text{tot}})^2} \right) \quad (39)$$

where the first integral is the contribution of the Type III states, the second (third) integral is that of the Type I states for which  $\mathbf{B}_{\text{eff}}$  is antiparallel (parallel) to  $\mathbf{B}$ . The limits of integration where  $N_{\text{tot}} = \infty$  correspond to  $\mathbf{B}_{\text{eff}}$  passing through zero. Evaluation of (39) using the relation (38) between  $n_c$  and  $n'_c$  once again yields the same Nyquist expression

$$S(f) = \frac{4e^2}{\beta h} \frac{n_b}{(m_b n_b + 1)} \quad (40)$$

as was obtained above by considering the simpler model with *only* Type III composite fermion Landau levels. Thus although the silent Type I levels contribute to the Johnson-Nyquist noise, remarkably, the *total* of the Type I and Type III Johnson-Nyquist noise is the same whether the Type I levels are present in the model or not.

The Johnson-Nyquist noise on quantum Hall plateaus for the case shown in Fig.1(b) where  $\mathbf{B}_{\text{eff}}$  is antiparallel to  $\mathbf{B}$  far from the edges can be calculated similarly. Here the Landau level filling in the bulk is the Jain fraction  $\nu = n_b/(m_b n_b - 1)$  with  $n_b$  a positive integer and  $m_b$  an even positive integer. In this case *only* the Type I levels contribute to the Johnson-Nyquist noise and the result of evaluating Equation (36) is

$$S(f) = \frac{4e^2}{\beta h} \frac{n_b}{(m_b n_b - 1)} \quad (41)$$

once again in agreement with the Nyquist formula  $S(f) = 4k_B T G$ . The result for  $S(f)$  is once again the same whether only the states with  $m = m_b$  or both the states with  $m = m_b$  and  $m = m_b + 2$  are included in the model.

## V. CONCLUSIONS

In this paper it has been shown that interactions between composite fermions must be included in composite fermion theories of electric current noise.

The important interactions were found to be those mediated by the fictitious electric field generated by composite fermion current fluctuations. Their effects must be calculated self-consistently. This was done analytically in mean field theory for systems with smooth edge potentials.

The interactions were found to renormalize the current fluctuations and the charges of the quasi-particles associated with composite fermions, making the quasiparticles fractionally charged. Analytic expressions were obtained for the charges of the quasiparticles in

both the incompressible and compressible regions of composite fermion systems. In the incompressible regions the calculated fractions agree with previous theories. In the compressible regions the quasiparticle charge varies smoothly with position. It tends to zero where the effective magnetic field vanishes and becomes equal to the electron charge where the composite fermion density becomes zero.

This theory of composite fermion current fluctuations and quasi-particle charge was then applied to an observable phenomenon by calculating the Johnson-Nyquist noise on fractional quantum Hall plateaus where there is no back-scattering. The results obtained were in agreement with the Nyquist formula  $S = 4k_B T G$ .

The Nyquist formula is usually derived from the fluctuation-dissipation theorem<sup>44</sup> so that this result is not surprising. However the present derivation of the Nyquist formula is of interest because it provides a good test of the mean field theory of current fluctuations and quasiparticle charge in this *interacting* system, and because it provides significant insights into the relationship between quasi-particle fractional charge, composite fermion edge states and current noise. Some of these are:

The Johnson-Nyquist noise in the fractional quantum Hall regime arises from *compressible* regions of the 2DEG. It is therefore *not* primarily due to quasi-particles having the usually quoted values of the fractional charge that correspond to incompressible fractional quantum Hall states. For example for the  $\nu = 1/3$  quantum Hall state the Johnson-Nyquist noise arises from quasiparticles whose charges are for the most part *not* equal to  $1/3$  of the electron charge but span a wide range of values.

Compressible edge channels associated with both the normal Type III and silent Type I composite fermion Landau level edge states<sup>4</sup> contribute to the Johnson-Nyquist noise when they are present at the edge, and their contributions sum to yield the Nyquist formula.

Shot noise differs from Johnson-Nyquist noise in that a finite voltage is applied to the sample and a non-zero average current flows. *Both* incompressible and compressible regions at the edge contribute to the current noise in this case. A full discussion of shot noise is beyond the scope of this paper, however it is already evident from the theory developed above that quasi-particles with a range of different values of the quasi-particle charge associated with compressible and incompressible strips at the sample edges will contribute to shot noise. Thus the present work suggests that the arguments that have been used recently<sup>26,27</sup> to infer a *single* value of the fractional quasi-particle charge from shot noise measurements may need some refinement.

This work was supported by NSERC of Canada.

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- <sup>41</sup> Note that the divergence of the composite fermion effective mass where  $\mathbf{B}_{eff} = 0$  complicates the interpretation and solution of Equation (6). See the Appendix of Ref.<sup>4</sup> for a discussion.
- <sup>42</sup> The integrand is appreciable over a range of states  $|r, k\rangle$  that are within a few magnetic lengths of the state  $|R, K\rangle$  in real space.  $L$  is considered to be large enough that there are many single-particle states in this range.
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## FIGURES

FIG. 1. Schematic drawing of the composite fermion Landau level structure near an edge. The effective magnetic field  $\mathbf{B}_{eff}$  well away from the edge is parallel to the real magnetic field in (a) and anti-parallel in (b). The apex of each “fan” of energy levels occurs where  $\mathbf{B}_{eff} = 0$  for an even integer  $m$ , where  $m$  is the number of fictitious flux quanta attached to each electron.  $m_b$  is the number of fictitious flux quanta attached to each electron in the bulk, far from the edges of the sample. Vertical dotted lines delimit the regions with different  $m$ . The different types of composite fermion edge states are labeled I, II and III.

FIG. 2. Schematic of a 2DEG of length  $L$  connecting source and drain reservoirs in a magnetic field. The reservoirs are effectively short circuited by the capacitors  $C$  and  $C'$  at the frequencies of interest. Arrows indicate the direction of propagation of edge states.

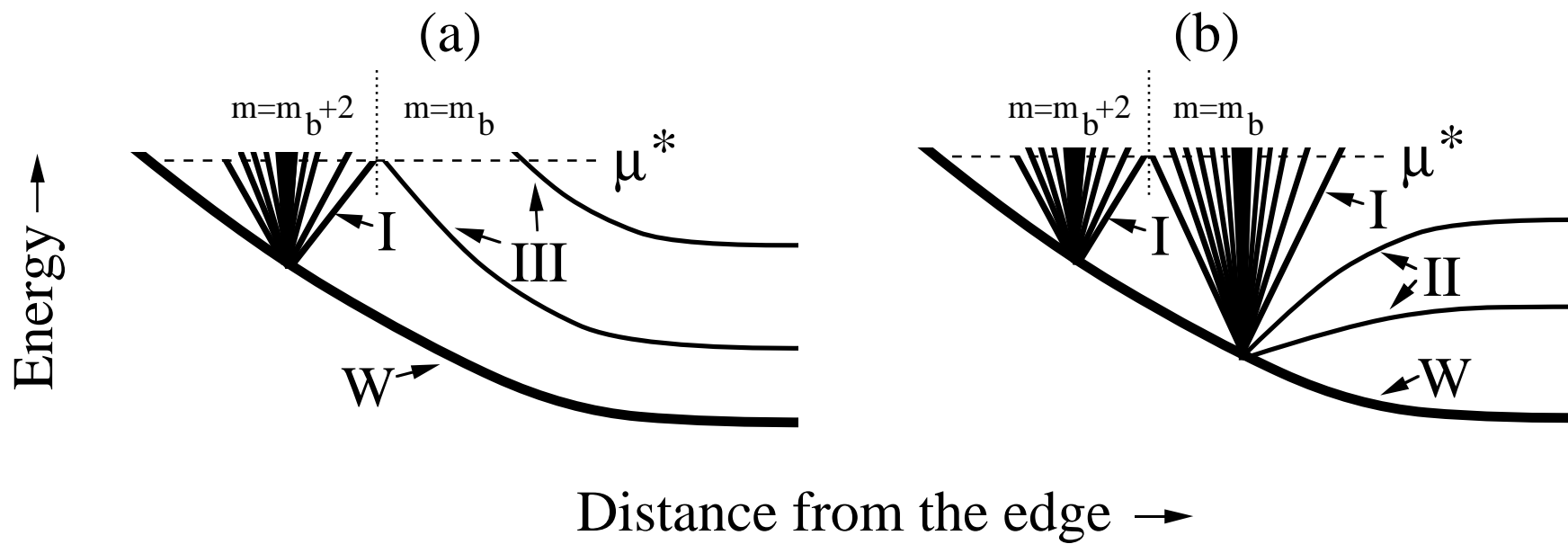


Figure 1  
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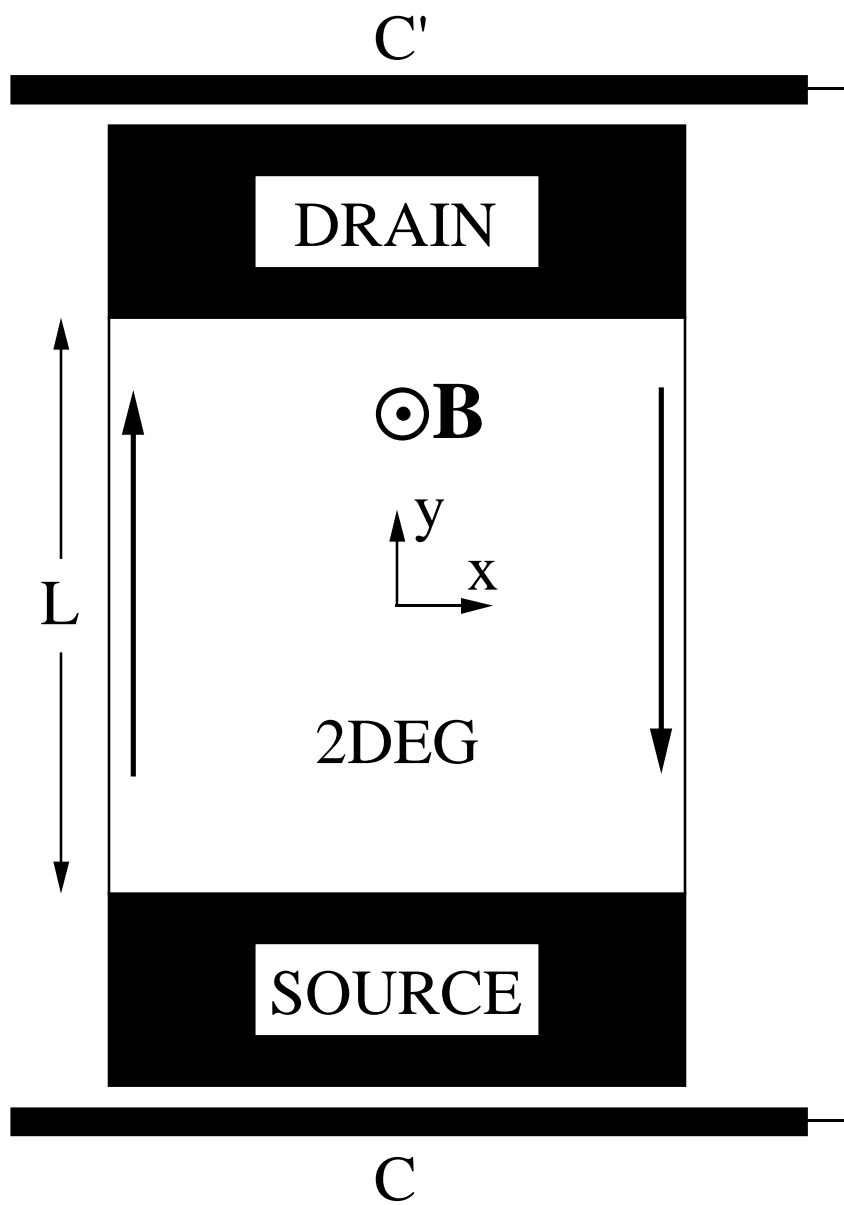


Figure 2  
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